



CANDIDATE  
NAME

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CENTRE  
NUMBER

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CANDIDATE  
NUMBER

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**4037/22**

October/November 2023

**2 hours**

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series*      $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series*      $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

**2. TRIGONOMETRY***Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

*Formulae for  $\triangle ABC$* 

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) A straight line passes through the points  $(4, 23)$  and  $(-8, 29)$ . Find the point of intersection,  $P$ , of this line with the line  $y = 2x + 5$ . [5]

- (b) Find the distance of  $P$  from the origin. [2]

- 2 Find the non-zero value of  $k$  for which the line  $y = -2x - 6k - 1$  is a tangent to the curve  $y = x(x + 2k)$ .  
[5]

**3 DO NOT USE A CALCULATOR IN THIS QUESTION.**

A cylinder has base radius  $(2 + \sqrt{3})$  m and volume  $\pi(16 + 9\sqrt{3})\text{m}^3$ . Find the exact value of its height, giving your answer in its simplest form. [4]

4 Solve the following equations.

(a)  $\frac{(e^{x+1})^2}{\sqrt{e^x}} = 10$

[4]

(b)  $2 \log_9 y - \log_9 (4y - 9) = \frac{1}{2}$

[5]

5 (a) Find the equation of the normal to the curve  $y = x^3 - 7x^2 + 12x - 5$  at the point  $(1, 1)$ . [5]

(b) Find the  $x$ -coordinates of the two points where the normal cuts the curve again. Give your answers in the form  $x = a \pm \sqrt{b}$  where  $a$  and  $b$  are integers. [5]

6 Find the exact value of  $\int_2^3 \frac{(x+2)^2}{x} dx$ .

[6]



- 7 A particle is travelling in a straight line. Its displacement,  $s$  metres, from the origin at time  $t$  seconds is given by  $s = 1.5e^{2t} + 2e^{-2t} - t$ .

(a) Find expressions for the velocity,  $v \text{ ms}^{-1}$ , and acceleration,  $a \text{ ms}^{-2}$ , of the particle. [3]

(b) Find the time,  $T$  seconds, when the particle is at rest. [4]

(c) Find the acceleration of the particle at time  $T$  seconds. [2]

8 A curve has equation  $y = x \sin 2x$ .

(a) Find  $\frac{dy}{dx}$ . [2]

(b) Find the equation of the tangent to the curve at  $x = \frac{\pi}{4}$ . [3]

- (c) Use your answer to **part (a)** to find the exact value of  $\int_0^{\frac{\pi}{6}} 2x \cos 2x dx$ . [5]

- 9 (a) An arithmetic progression has twelve terms. The sum of the first three terms is  $-36$  and the sum of the last three terms is  $72$ . Find the first term and the common difference. [5]

- (b) The first three terms of a geometric progression are 1, 1.2 and 1.44. Find the smallest value of  $n$  such that the sum of the first  $n$  terms is greater than 500. [5]

- 10 (a) By writing  $\cot x$  and  $\tan x$  in terms of  $\cos x$  and  $\sin x$ , show that

$$\frac{\sin x}{1 - \cot x} + \frac{\cos x}{1 - \tan x} = \sin x + \cos x. \quad [5]$$

- (b) Solve the equation  $9 \cot x + 3 \operatorname{cosec} x = \tan x$ , for  $0^\circ < x < 360^\circ$ . [5]

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